Workshop Cuntz Semigroups 2022

Kiel University, September 19-23, 2022.

Titles and Abstracts

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Ramon Antoine: *Traces on ultrapowers of C*-algebras*

Abstract: Every sequence of traces on a C*-algebra A induces a limit trace on a free ultrapower of A. Using Cuntz semigroup techniques, we characterize when such limit traces (and more generally limit quasitraces) are dense in the space of all (quasi)traces on a free ultrapower of A. As an application of our results, density of limit quasitraces on a free ultrapower implies that the lower semicontinuous functions are dense in the set of all dimension functions, thus verifying a conjecture by Blackadar and Handelman in this setting. Quite unexpectedly, we also obtain as another application that every simple C*-algebra that is (m, n)-pure in the sense of Winter is already pure.

Dawn Archey: *Permanence of Structural properties when taking crossed products.*

Abstract: Structural properties of C*-Algebras such as Stable Rank One, Real Rank Zero, and radius of comparison have played an important role in classification. Crossed product C*-Algebras are useful examples to study because knowledge of the base Algebra can be leveraged to determine properties of the crossed product. In this talk we will discuss the permanence of various structural properties when taking crossed products of several types. Crossed products considered will include the usual C* crossed product by a group action along with generalizations such as crossed products by a partial automorphism.

This talk is based on joint work with Julian Buck and N. Christopher Phillips and on joint work with Maria Stella Adamo, Marzieh Forough, Magdalena Georgescu, Ja A Jeong, Karen Strung, and Maria Grazia Viola.

Tristan Bice: *Hereditary C*-Subalgebra Lattices.*

Abstract: Thanks to foundational work of Rørdam, Ortega and Thiel, we know that Cuntz equivalence can be viewed as a relation between open projections or, equivalently, the corresponding hereditary C*-subalgebras. While the usual operation in the Cuntz semigroup is motivated by K-theory and sums of vector bundles, open projections also have a natural lattice structure where meets and joins directly extend the usual intersection and union of open sets. In our talk we will outline some of our previous research with Akemann on various connections between the algebraic structure of the C*-algebra and the lattice structure of its hereditary C*-subalgebras.

Joan Bosa: *Stable Elements and Property (S)*

Abstract: We study the relation (and differences) between stability and Property (S) in the simple and stably finite framework. This leads us to characterize stable elements in terms of its support, and study these concepts from different sides: hereditary subalgebras, projections in the multiplier algebra and order properties

in the Cuntz semigroup. We use these approaches to show both that cancellation at infinity on the Cuntz semigroup just holds when its Cuntz equivalence is given by isomorphism at the level of Hilbert right-modules, and that different notions as Regularity, ω-comparison, Corona Factorization Property, property R, etc.. are equivalent under mild assumptions.

Laurent Cantier: *On the Classification of Unitary Elements of a C*-Algebra.*

Abstract: The one-to-one correspondance between positive elements of a C*-algebra A and *-homomorphisms from $C_0(]0,1])$ to A has allowed to use the Cuntz semigroup to classify such morphisms, whenever the target algebra has stable rank one. This fact is the stepping stone that led to the complete classification by Robert of unital C*-algebras that can be written as inductive limits of building blocks of 1-dimensional NCCW-complexes that have trivial K_1-groups (e.g. AI algebras).

It is only natural to ask what happens if we look at unitary elements instead of positive elements? (Equivalently, if we look at *-homomorphisms with domain $C(\mathbb{T})$ instead of $C_0(]0,1])$.) We will see that this open problem cannot be solved by using similar methods as in the interval case, even in the most easiest settings and we will give some insight on how one can approach the case of the circle.

Jamie Gabe: *Semifinite tracial ultraproducts*

Abstract: I will introduce ultrapowers and ultraproducts of C*-algebras with respect to lower semicontinuous, densely defined tracial weights. In contrast to working with tracial states, these are not in general finite von Neumann algebras, but instead their multiplier algebras are semifinite von Neumann algebras. I will discuss applications to AF embeddability of the reduced C*-algebras of locally compact groups.

Shirly Geffen: *Pure C*-algebras of stable rank one.*

Abstract: The Cuntz semigroup has become increasingly important in connection with Elliott's classification program. Our work further contributes to the evidence that a C*-algebraic dimension type property, so called stable rank one, is deeply connected to purely algebraic properties of the Cuntz semigroup. In particular, we can show that, under natural assumptions, stable rank one can be recovered from almost divisibility and almost unperforation.

This is joint work in progress with Wilhelm Winter.

Ilan Hirshberg: *Dimension theory for non-free discrete group actions and the nuclear dimension of their crossed products.*

Abstract: I will discuss a few dimension-type invariants for discrete group actions on locally compact Hausdorff spaces, which are not assumed to be free, the most significant of which is something we call the long and thin covering dimension. Those are used to show that the associated crossed product has finite nuclear dimension. The result applies to arbitrary actions of finitely generated virtually nilpotent groups on finite dimensional spaces, but also covers other classes of actions, such as hyperbolic groups acting on the Gromov boundary. This generalizes

and puts in a more conceptual concept previous work of ours on non-free actions of the integers.

This is joint work in preparation with Jianchao Wu.

Bhishan Jacelon: *Genericity in the classifiable category*

Abstract: I will try to use different aspects of Genericity to thematically tie together a few threads of recent and ongoing work. In the sense of continuous model theory (or metric enriched category theory, if you prefer), a 'generic object' is in other words a Fraïssé limit, perhaps the simplest C*-example of which is the universal UHF algebra Q. In work that runs parallel to Masumoto's description of the Jiang-Su algebra $\mathcal{Z}$ as such a limit, but is unsurprisingly more complicated by the loss of the unit, Alessandro Vignati and I similarly construct the stably projectionless algebras W and Z_0. A major motivator of ours was to try to demonstrate extreme amenability of the automorphism groups of these various C*-algebras, but apart from Q, this remains an open problem. The second sense of 'generic object' I will discuss is measure theoretic, that is, an element of an almost-sure subset of a probability space. Random walks can be used to build UHF algebras, and one can appeal to classical theory for conditions under which Q is built with probability one. I will talk about this and other examples of C*-constructions associated with random (walks on) graphs, and what one might be able to say about the distributions of invariants like the tracial simplex, radius of comparison or K-theory. The final sense of 'generic object' I will touch on is topological, that is, an element of a residual subset of a topological space. In particular, Szabo, Wu and Zacharias established finite Rokhlin dimension as a generic property of automorphisms of separable, $\mathcal{Z}$-stable C*-algebras. I will show how (assuming the UCT) one can use this, together with the Elliott-Niu/Gong-Lin classification by traces of the class K of simple, separable, KK-contractible C*-algebras of finite nuclear dimension, to describe any such algebra up to stable isomorphism as the crossed product by a single automorphism of an algebraically simple element of K whose space of tracial states is a Bauer simplex with extreme boundary a Cantor space. Time permitting, I will indicate how similar ideas can be used to establish tracial chaos as a generic property of automorphisms of certain classifiable C*-algebras.

N. Christopher Phillips: *The dynamical radius of comparison.*

Abstract: This is joint work with M. Ali Asadi-Vasfi, and is work in progress: not everything below has been completely checked.

The Cuntz semigroup $\mathrm{Cu}(A)$ has a key place in the classification program for simple separable nuclear unital C*-algebras satisfying the Universal Coefficient Theorem. It is conjectured, and known in many cases, that if A is such a C*-algebra and $\mathrm{Cu}(A)$ has strict comparison, then A is classifiable in the sense of the Elliott program. The radius of comparison $\mathrm{rc}(A)$ is a numerical measure of the failure of strict comparison.

A dynamical Cuntz semigroup $\mathrm{Cu}_G(A)$, for an action of a discrete group G on a C*-algebra A, has been introduced by Bosa, Perera, Wu, and Zacharias, with the hope that it will play a similar role in the classifiability of crossed products. We introduce and study a corresponding radius of comparison $\mathrm{rc}_G(A)$. If G is finite (and for some results more generally), assuming a suitable weak version of approximate representability we get $\mathrm{rc}_G(A) = \mathrm{rc}(A)$, and assuming a suitable weak

version of the Rokhlin property we get

$$\mathrm{rc}_G(A) \leq \mathrm{rc}(C^*(G,A)) \leq \frac{\mathrm{rc}(A)}{\mathrm{card}(G)}.$$

Moreover, we exhibit many examples of actions on simple separable nuclear unital C*-algebras with "intermediate" behavior:

$$\frac{\mathrm{rc}(A)}{\mathrm{card}(G)} < \mathrm{rc}_G(A) < \mathrm{rc}(A).$$

Mikael Rørdam: *Inclusions of C*-algebras: Popa's averaging property for automorphisms.*

Abstract: Popa has introduced an averaging property of automorphisms which is very handy when studying inclusions of C*-algebras arising from dynamical systems. I will explain what the property is about and for which automorphisms it holds, and I will review applications of having this property in the context of inclusions of C*-algebras.

Andrew Toms: *Homotopy groups of Cuntz classes.*

Abstract: We consider the set of positive elements in a C*-algebra with fixed Cuntz class and, under some conditions, compute its homotopy groups. We show that for a class of simple unital separable real rank zero C*-algebras including AF algebras and noncommutative tori, these groups vanish when the Cuntz class in question is not compact.

Eduard Vilalta: *The Global Glimm Problem.*

Abstract: The Global Glimm Property was introduced by Kirchberg and Rørdam in their study of purely infinite C*-algebras, and it is known that any C*-algebra satisfying it is nowhere scattered (that is, it has no nonzero elementary ideal-quotients). The Global Glimm Problem asks the converse: Does every nowhere scattered C*-algebra have the Global Glimm Property?

In the talk, I will explain how both of the notions appearing in the Global Glimm Problem can be characterized by divisibility notions in the Cuntz semigroup. Using such characterizations, one can provide a new approach to the problem by introducing two conditions in the Cuntz semigroup that capture precisely what a nowhere scattered C*-algebra needs to have in order to satisfy the Global Glimm Property.

The talk is based on joint work with Hannes Thiel.

Stuart White: *Tracially complete C*-algebras.*

Abstract: Tracially complete C*-algebras are a new framework for studying the uniform trace norm completions of C*-algebras which arise in the Toms-Winter conjecture and the classification programme and mix both von Neumann algebraic and C*-algebraic phenomena. I'll describe these objects and their structure and

classification results, indicating how they interact with the analogous results for C* and von Neumann algebras.

This is joint work with José Carrión, Jorge Castillejos, Sam Evington, Jamie Gabe, Aaron Tikuisis and Chris Schafhauser.

Wilhelm Winter: *TBA.*

Abstract: TBA

Joachim Zacharias: *Almost Elementary C*-Dynamics, $\mathcal{Z}$-stability and the Dynamical Cuntz Semigroup*

Abstract: Motivated by the Toms-Winter conjecture and Kerr's notion of almost finiteness for actions of amenable discrete groups on compact metric spaces, which may be regarded as a dynamical analogue of $\mathcal{Z}$-stability in this setting, we propose a generalisation of this concept to actions of discrete groups on general C*-algebras which we coin almost elementary actions. Our starting point is a generalisation of Kerr's notion of a castle which we define as a simultaneous approximation of the algebra and the action, up to an arbitrarily small remainder in a dynamically tracial sense. To measure smallness we develop a dynamical version of the Cuntz Semigroup. It turns out that many different natural smallness conditions are equivalent. In the case of no group action our condition is a weak form of being tracially AF or having tracial nuclear dimension 0. We can show that in this case almost elementariness is equivalent to $\mathcal{Z}$-stability for separable simple nuclear algebras, thus it maybe added as another equivalent condition to the Toms-Winter conjecture. Moreover, almost elementary actions lead to $\mathcal{Z}$-stable crossed products, in line with it being a kind of dynamical $\mathcal{Z}$-stability.

Joint work with Joan Bosa, Francesc Perera and Jianchao Wu